Optimal Storage Rack Design for a 3D Compact AS/RS with Full Turnover-Based Storage

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Optimal Storage Rack Design for a 3D compact AS/RS with full turnover-based storage

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Abstract

Compact, multi-deep (3D) automated storage and retrieval systems (AS/RS) are becoming increasingly popular for storing products with relatively low turnover on a compact area. An automated storage/retrieval crane takes care of movements in the horizontal and vertical direction in the rack, and a gravity conveying mechanism takes care of the depth movement. An important question is how to layout such systems to minimize the product storage and retrieval times. Although much attention has been paid to 2D AS/RS, multi-deep systems have hardly been studied. This paper studies the impact of system layout on crane travel time. We calculate the rack dimensions that minimize single-command cycle time under the full-turnover-based storage policy. We prove the expected travel time is minimized when the rack is square-in-time in horizontal and vertical directions and the conveyor's dimension is the longest. We compare the model's results with the performance of the random storage policy and show a significant crane travel time reduction can be obtained. We illustrate the findings of the study by applying them in a practical example.

Keywords: Order picking; Storage rack design; AS/RS; Travel time model; Warehousing; Turnover-based storage

1. Introduction

Compact, multi-deep (3D), automated storage/retrieval systems (AS/RS) become increasingly popular for storing products with relatively low unit-load demand, on standard product carriers. In principle, every load can be accessed individually, although some shuffling may be required. They are also used to automatically presort unit loads within the system, so that these loads can be retrieved rapidly when they are needed (Le-Duc et al. 2005). Their main advantage, besides the full automation of tasks, is the large storage capacity on a limited area, which can make such systems more cost efficient than traditional single-deep AS/RS. An example is the system of Miele in Gütersloh (D), where a combination of cranes and shuttles store and retrieve individual palletized white goods (like washing machines and dish washers), and automatically sequence them for loading trains and trailers. We discuss a new multi-deep AS/RS, with a major innovative contribution in its cheap construction. Potential application areas are also innovative. We have studied applications in dense container stacking at a container yard and the Distrivaart project in the Netherlands (Waals, 2005), where pallets are transported by barge shipping between several suppliers and several supermarket warehouses. This project has actually been implemented and has resulted in a fully automated storage system on a barge (see Figure 1).

<Insert Figure 1 here>

The rack in the system is sketched in Figure 2 and consists of a 3D storage rack, a depot (or I/O point), an S/R machine (or crane), and a gravity conveying mechanism with conveyors operating in pairs responsible for the depth movement in conjunction with an elevating mechanism. The same system has been studied by Le-Duc et al. (2005). The pallets enter and leave the system via the I/O point and are stored in the rack. The S/R machine can drive and lift simultaneously and takes care of the movements in the horizontal and vertical directions. It picks up pallets from the I/O point to bring them to a storage conveyor or retrieves them from a conveyor to bring them to the I/O point. The gravity conveyors work in pairs: on the inbound conveyor pallets flow to the back end of the rack by controlled gravity. At the back of the rack a simple, inexpensive elevator lifts the pallet to the neighboring outbound conveyor from which it flows to the front end of the rack. Consequently, unit loads can rotate independently of the S/R machine and can be retrieved individually.

<Insert Figure 2 here>

Different storage policies can be used to store pallets in the rack, in particular random and dedicated storage policies (including full-turnover-based policies). Under the random storage policy, each pallet is equally likely to be stored in any of the storage positions in the rack. In reality, usually some items are requested more frequently than others, as described by an ABC curve. Full turnover-based storage policies store pallets at a travel distance of the I/O point decreasing with increasing item turnover frequency, and thereby realize short storage/retrieval times.

Determining the throughput or, alternatively, the crane cycle time, is one of the main issues in designing a compact system. It depends on material handling speeds and capabilities, but also on system dimensions, crane dwell-point strategy, and storage and retrieval policies. While this problem has been tackled for 2D systems to a great extent, literature on 3D systems is far from abundant. We try to fill this gap by determining the expected S/R travel time for single command-storage or retrieval requests and determining the optimal rack dimensions. We assume a full turnover-based storage policy and compare our results with those of the random storage policy as studied by Le-Duc et al. (2005). We apply the full turnover-based storage policy in such a rack by sequentially assigning products on unit loads, sorted by decreasing turnover frequency of the unit loads to positions on the conveyor pairs that are sorted on increasing travel distance from the I/O point. For a given rack storage capacity, we express the single-command S/R machine travel time as a function of the rack dimensions. Due to the complexity of the model, we first give several theorems to reduce the feasible area of the model variables without losing the optimal solution and show the optimal structure of the rack must be square-in-time (SIT) and the conveyor's dimension the longest. With these results, the proposed model is simplified into an equivalent non-linear convex programming model that can be solved numerically to obtain the optimal solution. Finally the results of the model are compared with those of the random storage policy in Le-Duc et al. (2005), and a practical example is given to illustrate the application of our research. Optimal racks using a full turnover-based storage policy can obtain significant reductions (up to 68%, depending on the steepness of the ABC curve) in the expected travel time compared with random storage.

2. Literature review

Designing an efficient AS/RS has interested many researchers for decades. Performance measures used include travel time per S/R operation cycle, number of S/R requests carried out per unit time, and average cost per S/R operation. Much literature focuses on the travel time per S/R operation cycle which depends on the shape of the storage rack (ratio between different dimensions: SIT, or non-SIT (NSIT)), pallet storage policies (random/dedicated S/R policies), the S/R crane's operation modes (single, dual, and multiple commands per cycle), dwell point policies (at the middle or corner of the rack), and the number of rack dimensions (2D or 3D racks). Because this paper discusses how to dimension the 3D rack by minimizing the expected travel time for a single command cycle under the full turnover-based storage policy, in this section, we only review literature closely related to our research. We focus on travel time calculation with different rack shapes, on storage policies, and on 3D rack systems.

Storage rack shape. Calculating the travel time based on different rack shapes with Chebyshev travel has received considerable interests since the study of Hausman et al. (1976). They calculate the one-way travel time for a single command cycle based on a SIT-rack system with different storage policies: random, turnover, and class-based storage. Bozer and White (1984) obtain the travel time for single and dual command cycles for NSIT rack systems under the random storage policy, and prove that with a constant AS/RS speed, the SIT rack is the optimal 2D-rack configuration. In practice other rack shapes exist, given the various cost components as well as height and length constraints. Based on Bozer and White's travel time model for NSIT racks, Eynan and Rosenblatt (1994) develop a procedure for dividing a rectangular rack into storage classes and calculate the travel time resulting from class-based storage. Recently, travel time as main performance criterion is used in Pan and Wang (1996), Park et al. (2003), Hu et al. (2005), Park et al. (2006), and Park (2006) for different types of NSIT racks systems. In the above literature, only Bozer and White (1984) take the travel time as a function of the rack dimensions, and minimize the travel time by dimensioning the 2D rack.

Storage policies. Under the random storage policy S/R requests are allocated randomly over the available storage locations in a rack. This method is considered widely in the literature, see for example Bozer and White (1984), Lee and Elsayed (2005), and Le-Duc et al. (2005). In many studies, like Hausman et al. (1976) and Lee and Elsayed (2005), it is used to benchmark improvements of other storage policies. The full turnover-based policy was first described by Heskett (1963, 1964) as the Cube-per-Order index (COI) rule without a proof of its optimality. Kallina and Lynn (1976) discuss the implementation of the COI rule in practice. The earlier mentioned work of Hausman et al. (1976) assumes a Pareto (or ABC)-demand curve and a basic EOQ (Economic Order Quantity)-based reordering policy, in their derivation of an expression for the expected single-command travel time for random and full turnover-based storage. Graves et al. (1977) extend this to an expression for the expected dual-command travel time under these storage policies. These analytical results under the full turnover-based storage policy are derived for SIT racks. The formulation by Hausman et al. (1976) to calculate the one way travel time is a universal expression which can be used for NSIT racks and multi-deep racks as well, because in its derivation only EOQ assumptions and an ABC-demand function are used. It has been used by other researchers in different warehouse settings. For example, Koh et al. (2002) apply it to estimate the travel time for a warehousing system with a crane in combination with a carousel. Kim and Seidmann (1990) assume a product turnover distribution function different from that of Hausman et al. (1976) resulting in a different single-command travel time function. Their turnover distribution function has the advantage that it is analytically more tractable. However, it has the disadvantage that it is not concave as Figure 1 in their paper shows. For NSIT racks, Park et al. (2003) assume that the full turnover-based distribution function is given as $G(x) = x^s$ for $0 < x \leq 1$ and $0 < s \leq 1$, identical to Hausman et al. (1976), but they assume that every item type has only one pallet in the storage rack which is not appropriate in many cases. The same full turnover-based distribution function is also used by Park (1999), Park (2006), and Park et al. (2006).

3D rack systems. Park and Webster (1989a) propose a conceptual model that can help a warehouse planner in the design of certain 3D pallet-storage systems by minimizing the total storage system costs. The costs consist of land, building, handling equipment, storage-rack, labor, maintenance, and operating costs. Park and Webster (1989b) deal with a "cubic-in-time" layout, for minimizing the travel time of selected handling equipment. In these two publications, the rack dimensions are given or, in other words, the problem of determining the optimal rack dimensions is neglected. Sari et al. (2005)

study a 3D flow-rack AS/RS where the pallets are stored and retrieved at different rack sides by two cranes. In order to retrieve a particular pallet, the retrieval crane has to move all pallets in front of it and store these on a special restoring conveyer. They derive the travel time for the random storage policy with given lengths of the three rack dimensions. Le-Duc et al. (2005) extend the method of Bozer and White (1984) for 2D rack systems to three dimensions and find the optimal design of the 3D rack system by minimizing the expected single- and dual-command travel time of the S/R machine of random S/R requests under random storage policies. They conclude that the optimal ratio of the three dimensions in vertical, horizontal and conveyor directions is $0.72:0.72:1$ for single-command systems and 0.84:0.84:1 for dual-command systems.

No literature exists on travel time estimation and/or optimal system dimensioning for 3D AS/RS with the full turnover-based storage policy. In the following sections, we will step by step estimate the single-command travel time of the S/R machine after first introducing the problem assumptions.

3. Assumptions and general model

3.1 Assumptions

The studied system is identical to that of Le-Duc et al. (2005), and sketched in Figure 2. We follow the assumptions of Le-Duc et al. (2005) (see also Hausman et al. 1976, Bozer and White 1984, 1990, 1996 Ashayeri et al. 2002, Foley et al. 2004)

- The 3D rack is considered to have a continuous rectangular pick face, where the I/O point (or depot) is located at the lower left-hand corner of the rack (see Figure 2). When the crane is idle, it stops at the I/O point
- The S/R machine (or crane) is capable of simultaneously moving in vertical and horizontal direction at constant speeds. Thus, the travel time required to reach any location in the rack (or a storage conveyor pair in our case) is represented by the Chebyshev metric.
- The conveyor can move loads in an orthogonal depth direction, independent of the S/R machine movement, at a constant speed.
- The S/R machine operates on a single-command basis (multiple stops in the aisle are not allowed).
- Each pallet holds only one item type. All storage locations and pallets have the same size. Therefore all storage locations can be used for storing any pallet load. The items are replenished according to the EOQ model.
- Following Hausman et al. (1976), we assume the pick-up/deposit (P/D) time for the crane to pick up or deposit a pallet can be ignored. This is justified if the P/D time is fairly small compared to the total crane travel time.
- We use a full turnover-based storage policy. That is, the storage position of each pallet is determined by its relative activity among all pallets in the rack by sorting the pallets from most to least active, starting from the I/O point. One item type can have multiple pallets.

3.2 Notations and general model

The length (*L*), the height (*H*) of the rack, and the perimeter (of length 2*P*) of the conveyor form three orthogonal dimensions of the rack, in which the speeds of the conveyor, and the S/R machine's speed in the horizontal and vertical direction are s_c, s_h, s_v respectively.

To standardize the system, we define the following quantities.

$t_c = \frac{2*P}{s_c}$: length (in time) of the conveyor.

$t_h = \frac{L}{s_h}$: length (in time) of the rack.

$t_v = \frac{H}{s_v}$: height (in time) of the rack.

$T = \max\{t_h, t_v, t_c\}$

$b = \min\left\{\frac{t_h}{T}, \frac{t_v}{T}, \frac{t_c}{T}\right\}$. Note that $0 < b \le 1$ and $b = 1$ if $t_h = t_v = t_c$.

a is the remaining element (besides b and 1) of the set $\left\{\frac{t_h}{T}, \frac{t_v}{T}, \frac{t_c}{T}\right\}$, thus $0 < b \le a \le 1$.

For determining the optimal dimensions of the rack, we suppose that $H*L*P$ is a constant. As a result $t_h t_v t_c = V$ (the storage capacity in cubic time) is also a positive constant.

Assume that a S/R request location is represented by (x, y, z), and X, Y and Z refer to the movement directions of the S/R machine or conveyor: the longest dimension refers to the Z direction, the shortest dimension refers to the Y dimension and the left medium dimension refers to the X direction. We can see that the S/R machine's travel time for single-command cycles (ESC) consists of the following components:

- Time needed to go from the depot to the pickup or drop-off position and to wait for the load or empty slot to be available at the pick/drop position (if the conveyor's circulation time is larger than the travel time of the S/R machine), W. In other words, W is Chebyshev metric, and the maximum of the following three quantities:
  - time needed to travel horizontally from the depot to the pick/drop position,
  - time needed to travel vertically from the depot to the pick/drop position,
  - time needed for the conveyor to circulate the load or empty slot from the current position to the pick/drop position.
- Time needed for the S/R machine to return to the depot from the request point, U.

Hence, the expected travel time can be expressed as follows:

$$ESC = E(W) + E(U). \tag{1}$$

In order to calculate ESC under the full turnover-based policy, we recall Hausman et al. (1976). In their paper, in order to calculate the turnover of each item in a storage space, they model the well-known ABC curve as

$$G(i) = i^{\delta}, \quad 0 < \delta \le 1, \tag{2}$$

where i is the percentage of inventoried items, $0 < i \le 1$, δ is the skewness of the ABC curve, and $G(i)$ is the cumulative percentage of demand in full pallet loads. Under the full turnover-based storage policy, for a fraction or percentage *i* of the items, they derive the expected one-way travel time for the crane traveling from the I/O point to a random P/D position of a request to pick-up or store a pallet (denoted by $T_T^{'}$ in their paper) as:

$$T_T^{'} = \frac{\int_{j=0}^{1} \lambda(j) y(j) dj}{\int_{j=0}^{1} \lambda(j) dj}, \tag{3}$$

In which $\lambda(j)$ is the turnover, of the jth pallet in the rack, and

$$\lambda(j) = (\frac{2\delta}{K})^{1/2} j^{(\delta-1)/(\delta+1)}, \quad 0 < j \leq 1, \tag{4}$$

where K is the ratio of order cost to holding cost which is assumed to be identical for all items. $y(j)$ is the ranked one-way time for the crane to travel from the I/O point to location j and $0 < y(j) \leq 1$, where by definition the jth percentile of the locations is closer to the I/O point than the location under consideration. For 2D SIT racks, $y(j)$ equals $j^{1/2}$ (Hausman et al. 1976).

In order to calculate $E(W)$, $y(j)$ needs to be calculated in non-cubic in time setting. The calculation of $y(j)$ is given in Appendix A, from which we have

$$y(j) = \begin{cases} (abj)^{1/3} & 0 < j \leq b^2/a \\ (aj)^{1/2} & b^2/a < j \leq a \\ j & a < j \leq 1 \end{cases}. \tag{5}$$

Substituting (5) into (3), and multiplying the result with T results in

$$E\left(W\right) = T \times \frac{\int_{j=0}^{b^2/a} \lambda(j) \sqrt[3]{abj}\, dj + \int_{j=b^2/a}^{a} \lambda(j) \sqrt{aj}\, dj + \int_{j=a}^{1} \lambda(j) j dj}{\int_{j=0}^{1} \lambda(j) dj}$$

$$\Rightarrow E\left(W\right) = T\left(\frac{sb^{2s+1}}{(2s+1)(3s+1)a^s} + \frac{sa^{s+1}}{(s+1)(2s+1)} + \frac{s}{s+1} \right), \tag{6}$$

where $s = 2\delta/(1+\delta)$.

$E(U)$ can be obtained in a similar fashion, by neglecting the depth movement.

Without loss of generality, we suppose $t_h \geq t_v$. Set $\beta = t_v / t_h$ which is the rack shape in the crane's moving directions. If we standardize $t_h = 1$, then similar to the above procedure for obtaining (5), we have

$$y(j) = \begin{cases} (\beta j)^{1/2} & 0 < j \leq \beta \\ j & \beta < j \leq 1 \end{cases}. \tag{7}$$

Substituting (7) into (3) and multiplying the result with T, $E(U)$ is obtained as

$$E(U) = \frac{\int_{j=0}^{\beta} \lambda(j)\sqrt{\beta j}\,dj + \int_{j=\beta}^{1} \lambda(j)\, j\,dj}{\int_{j=0}^{1} \lambda(j)\,dj} t_h = \frac{s(2s+1+\beta^{s+1})}{(s+1)(2s+1)} t_h, \tag{8}$$

More details on the calculation of $E(W)$ and $E(U)$ can be found in Yu and De Koster (2006).

From (1), (6) and (8), the mathematical model to determine the optimal pallet storage rack system then can be determined by the following general model (denoted as GM):

Model GM:

$$\begin{aligned} &Minimize \quad ESC(a,b,T) = E(U) + E(W) \\ &= \frac{s(2s+1+\beta^{s+1})}{(s+1)(2s+1)} t_h + \left(\frac{sb^{2s+1}}{(2s+1)(3s+1)a^s} + \frac{sa^{s+1}}{(s+1)(2s+1)} + \frac{s}{s+1} \right) T \\ &subject\ to \quad abT^3 = V \\ &\qquad \beta = \begin{cases} b/a & if\ t_c = T \\ b & if\ t_c = aT \\ a & if\ t_c = bT \end{cases} \\ &\qquad t_h = \begin{cases} aT & if\ t_c = T \\ T & if\ t_c = aT \\ T & if\ t_c = bT \end{cases} \end{aligned} \tag{9}$$

where $T > 0$ and $0 < b \leq a \leq 1$.

When the optimal values of variables a, b, T of model GM can be determined, the expected travel time is minimized for a given rack capacity V. T In order to find these optimal solutions, we distinguish the following three cases:

- The conveyor's length is the longest dimension (denoted by **CL**), or $t_c = T$ and $t_v : t_h : t_c \equiv b : a : 1$;
- The conveyor's length is the medium dimension (denoted by **CM**), or $t_c = aT$ and $t_v : t_h : t_c \equiv b : 1 : a$;
- The conveyor's length is the shortest dimension (denoted by **CS**), or $t_c = bT$ and $t_v : t_h : t_c \equiv a : 1 : b$.

4. Properties and equivalent model

Solving Model GM directly based on the three cases CL, CM, and CS is difficult. Therefore, we propose several theorems to simplify it. Theorems and 2 show that the cases CS and CM can be neglected respectively. Theorem 3 shows the optimal rack shape is SIT. These theorems lead to a much easier nonlinear convex programming model equivalent to model GM.

We first reformulate Model GM for the three cases: CL, CM and CS respectively.

For the case CL, $t_c = T$ and the corresponding model can be presented as:

$$\begin{aligned}
&\textit{Minimize} \;\; ESC_{CL}(a,b) = \frac{V^{1/3} a^{-s} s}{(1+s)(1+2s)(1+3s)(ab)^{1/3}} (a^s + a^{1+s} + a^{1+2s} + b^{1+s} \\
&\qquad + b^{1+2s} + 5a^s s + 5a^{1+s} s + 3a^{1+2s} s + 3b^{1+s} s + b^{1+2s} s + 6a^s s^2 + 6a^{1+s} s^2) \quad (10) \\
&\textit{subject to} \quad 0 < b \le a \le 1.
\end{aligned}$$

For the case CM, $t_c = aT$ and the corresponding model turns out to be:

$$\begin{aligned} \textit{Minimize}\ \ & ESC_{CM}(a,b) = \frac{V^{1/3}a^{-s}s}{(1+s)(1+2s)(1+3s)(ab)^{1/3}}(2a^{s} + a^{1+2s} + a^{s}b^{1+s} \\ & \quad + b^{1+2s} + 10a^{s}s + 3a^{1+2s}s + 3a^{s}b^{1+s}s + b^{1+2s}s + 12a^{s}s^{2}) \qquad (11) \\ \textit{subject to}\ \ & 0 < b \le a \le 1. \end{aligned}$$

For the case CS, $t_c = bT$ and the corresponding model can be presented as:

$$\begin{aligned} \textit{Minimize}\ \ & ESC_{CS}(a,b) = \frac{V^{1/3}a^{-s}s}{(1+s)(1+2s)(1+3s)(ab)^{1/3}}(2a^{s} + 2a^{1+2s} + b^{1+2s} \\ & \quad + 10a^{s}s + 6a^{1+2s}s + b^{1+2s}s + 12a^{s}s^{2}) \qquad (12) \\ \textit{subject to}\ \ & 0 < b \le a \le 1. \end{aligned}$$

We denote (a_l, b_l), (a_m, b_m), and (a_s, b_s) as the optimal variable values of Models (10), (11), and (12) where the minimal objective function values are denoted by $ESC^{*}_{CL}(a_l, b_l)$, $ESC^{*}_{CM}(a_m, b_m)$, and $ESC^{*}_{CS}(a_s, b_s)$, respectively.

The optimal variable value of (a,b), denoted by (a^{*}, b^{*}), of Model GM satisfies

$$(a^{*}, b^{*}) = \arg\min_{a,b}\{ESC^{*}_{CS}(a_s, b_s),\ ESC^{*}_{CM}(a_m, b_m),\ ESC^{*}_{CL}(a_l, b_l)\}. \qquad (13)$$

The minimum objective value of Model GM is $ESC^{*}(a^{*}, b^{*}, T^{*})$ where $T^{*} = V^{1/3}/(a^{*}b^{*})^{1/3}$.

4.1 Simplifying Model GM

Theorem 1. The minimal objective function value $ESC^{*}_{CM}(a_m, b_m)$ of model (11) is

(I) equal to the minimal objective function value $ESC^{*}_{CS}(a_s, b_s)$ of model (12) if $a_s = b_s = a_m = b_m$.

(II) less than the minimal objective function value $ESC^{*}_{CS}(a_s, b_s)$ otherwise.

Proof. See Appendix B.

Case I, or $a_s = b_s$ in Theorem 1, represents the situation where the optimal rack's y-dimension (b_s) equals the x-dimension (a_s) in the CS case. This rack configuration is included in the CM case if $a_m = b_m$ (note that (a, b) represent different dimensions in the two cases). We conclude from Theorem 1:

"*The case CS in Model GM can be neglected for calculating the optimal solution of Model GM.*"

Theorem 2 is similar to Theorem 1. We state it here without proof (this follows the same lines as the proof of Theorem 1). It shows the case CM can be neglected in calculating the optimal solution of Model GM.

Theorem 2. The minimal objective function value $ESC^{*}_{CL}(a_l, b_l)$ of model (11) is

(I) equal to the minimal objective function value $ESC^{*}_{CM}(a_m, b_m)$ of model (12) if $a_l = b_l = b_m = a_m$ =1 (i.e. cubic-in-time).

(II) less than the minimal objective function value $ESC^{*}_{CM}(a_m, b_m)$ otherwise.

From Theorem 1 and Theorem 2, we conclude:

"*All optimal solutions of Model GM exist in the case CL (i.e., Model (10)). The model (10) is an equivalent to Model GM.*"

It is obvious that finding the optimal expected travel time of Model (10) is easier than Model GM. However this is still fairly complicated as its objective function cannot be proven to be convex or concave. We therefore have to analyze the problem further. With Theorem 3 we prove the optimal 3D rack must be SIT.

Theorem 3. For the 3D rack, the expected travel time with the full turnover-based storage policy will be minimized **only when** the rack is SIT and the conveyor's length is the longest.

Proof. See Appendix C.

4.2 The equivalent model of Model GM and its solution

From Theorem 3, we conclude the optimal solution of Model GM has the following properties: $T = t_c$, $a = b \left(\text{thus } \beta = 1\right)$, $t_h = t_v = at_c$, and $a^2 t_c^{\,3} = V$. Therefore a model equivalent to model GM is the following constrained-optimization problem:

$$\begin{aligned} &\textit{Minimize} \quad ESC(a) = \frac{V^{1/3}s}{(1+s)(1+2s)(1+3s)(a)^{2/3}}(1+2a+2a^{1+s}+5s \\ &\qquad\qquad +8as+4a^{1+s}s+6s^2+6as^2) \qquad (14) \\ &\textit{subject to} \quad D = \left\{a \middle| 0 < a \le 1\right\}. \end{aligned}$$

Since $\dfrac{d^2ESC(a)}{da^2} = \dfrac{2sV^{1/3}}{9a^{8/3}(1+s)(1+2s)}(5 - 2a - 2a^{1+s} + 10s - 2as - a^{1+s}s + 6a^{1+s}s^2) > 0$ and constraint D is linear, the problem is a strict convex non-linear programming problem.

At this point, if the critical point a^* of equation $\dfrac{dESC(a)}{da} = 0$ is in D, we have found the minimum objective function value $ESC(a^*)$, where

$$\frac{dESC(a)}{da} = \frac{2sV^{1/3}}{3a^{5/3}(1+s)(1+2s)}(-1 + a + a^{1+s} - 2s + as + 2a^{1+s}s). \qquad (15)$$

Because $\lim\limits_{a\to 0}\dfrac{dESC(a)}{da} = -\infty$, $\dfrac{dESC(a)}{da}\Big|_{a=1} = 2V^{1/3}s/(3(1+2s)) > 0$, and $\dfrac{dESC(a)}{da}$ is continuous, the unique critical point a^* of equation $\dfrac{dESC(a)}{da} = 0$ must be in D.

Equation $\dfrac{dESC(a)}{da} = 0$ can be solved numerically for any given s. The optimal solution of Model (14) is given by the optimal decision variable $a = a^*$ and the optimal objective function value $ESC(a^*)$. a^* is a function of s, according to (15). Because $a^2 t_c^{\,3} = V$, we have $t_c^{\,*} = V^{1/3} / a^{*2/3}$. Again, because $t_h = t_v = at_c$, we have $t_v^{\,*} = t_h^{\,*} = (a^* V)^{1/3}$.

From the above analysis, we conclude the following for Model GM:

(a) Given a 3D rack with a total storage capacity V, the expected travel time of the S/R machine will be minimized if $t_v^* = t_h^* = (a^* V)^{1/3}$ and $t_c^* = V^{1/3} / a^{*2/3}$ (i.e. $t_v^* : t_h^* : t_c^* = a^* : a^* : 1$) and the optimal expected travel time for the single command cycle is $ESC(a^*)$ where $ESC(a)$ is the objective function of Model (14) and a^* is the solution of the equation $\frac{dESC(a)}{da}$ =0.

(b) The optimal ratio of the three dimensions $t_v^* : t_h^* : t_c^*$ is independent of the rack capacity $V > 0$.

5. Comparing the results with those of Le-Duc et al. (2005)

Le-Duc et al. (2005) consider randomized storage policies in which any point within the rack is equally likely to be selected for storage or retrieval. Their problem corresponds to $\delta = 1$ or $s = 1$ in our paper. Let $s = 1$, the objective function of Model GM turns into:

$$ESC(a,b,T) = \left(\frac{\beta^2}{6} + \frac{1}{2} \right) t_h + T \left(\frac{b^3}{12a} + \frac{a^2}{6} + \frac{1}{2} \right), \tag{16}$$

which is the same as that of Le-Duc et al. (2005). The optimal solution for a random storage policy can be found from Le-Duc et al. (2005) as: $t_h^* = t_v^* = 0.90V^{1/3}$, $T = t_c^* = 1.24V^{1/3}$ and ESC^*=$1.38V^{1/3}$.

In our paper, using the conclusion in Section 4.2, we can find the optimal solution and its expected travel time for the 3D AS/RS rack system for every skewness parameter value corresponding to a particular ABC curve. Using Equation (2) to represent an ABC curve, the notation $i / G(i)$ denotes that a fraction i of the inventoried items represents a fraction $G(i)$ of the total demand. For a given $i / G(i)$ combination we can obtain δ from Equation (2), by solving $\delta = \ln G(i) / \ln i$. According to the relationship between s and δ, the corresponding s can also be determined as $s = \delta / (2 - \delta)$.

Table 1 tabulates values of the optimal solutions for different $i / G(i)$ combinations and their corresponding s or δ values for a given V. In this table, the time values for t_h^*, t_v^*, t_c^* and ESC^* are expressed in the quantities $V^{1/3}$. In Figure 3, the expected travel time of our full turnover-based storage policy is compared with that of the random storage policy in Le-Duc et al. (2005), for various ABC curves, and shows the corresponding expected travel time improvement. In this Figure, Series "ESC_{FT}", "ESC_{RAN}" and "Time saved" represent the optimal ESC^* value of this paper, the optimal ESC^* value of Le-Duc et al. (2005), and the percentage improvement $(ESC_{FT} - ESC_{RAN}) / ESC_{RAN} \times 100\%$, respectively.

<Insert Table 1 here>

<Insert Figure 3 here>

From Table 1 and Figure 3, it can be seen that

(1) When $0 < \delta < 1$ (all cases except 20%/20%), reductions in the expected travel time are obtainable from the turnover-based storage policy compared with the random storage policy in the 3D rack system. The reduction percentage depends on the steepness of the ABC curve. For a 20%/90% ABC curve with $\delta = 0.07$, the improvement is significant and the percent travel time saved is 67.68%.

(2) When $\delta = 1$ (20%/20%), our result is the same as that of Le-Duc et al. (2005) with the random storage policy. The problem in their paper is a special case of that

in this paper with $\delta = 1$.

(3) For the turnover-based storage policy, the smaller the skewness parameter δ is in the ABC curve, the more sensitive the expected travel time is. For example, when δ decreases from 1 to 0.86 (20%/20% to 20%/30%), the relative ESC decreases $\frac{1.38V^{1/3} - 1.31V^{1/3}}{1-0.75} = 0.26V^{1/3}$, however when δ decreases from 0.24 to 0.12 (20%/80% to 20%/90%), the relative ESC decreases $\frac{0.72V^{1/3} - 0.45V^{1/3}}{0.14-0.07} = 3.74V^{1/3}$, which is much bigger than $0.26V^{1/3}$.

6. An example

As an illustrating example, assume that we have to design a 3D compact system with data as given in Table 2, based on those in Le-Duc et al. (2005). The layout of the system refers to Figure 2. The problem is to find the approximate optimal dimensions of the system so that the expected travel time is minimized for two ABC curves considered in Table 2.

<Insert Table 2 here>

The rack should have sufficient capacity to store 2000 pallets, which means the rack should have at least $V = 0.5 \times 0.5 \times 2.17 \times 2000$ =1085 (seconds^3).

Recalling the conclusion in Subsection 4.2, we obtain the optimal solutions for a continuous rack system: (1) for the 20%/20% ABC curve, $t_c^* = 1.24\sqrt[3]{V} = 12.78$ (seconds), $t_h^* =$ t_v^* $= 0.72t_c^*$ $= 9.21$ (seconds) and the optimal travel time ESC^* = $1.38\sqrt[3]{V}$ =14.20 (seconds); (2) for the 20%/90% ABC curve,

$t_c^* = 1.50\sqrt[3]{V} = 15.37$ (seconds), $t_h^* = t_v^* = 0.55 t_c^* = 8.40$ (seconds) and the optimal travel time $ESC^* = 0.45\sqrt[3]{V} = 4.59$ (seconds).

However, in a real-world setting, AS/RS systems are discrete and the rack dimensions must be integral multiples of the pallet dimensions. Therefore, we choose 'practical optimal' dimensions such that they are as close as possible to the corresponding optimal dimensions found while the system storage capacity is at least 2000 pallets. We obtain the following practical approximate optimal dimensions and expected travel time for both ABC curves: (1) for the 20%/20% ABC curve, $\overline{t_h}^* = 9$ seconds (18 pallets), $\overline{t_v}^* = 8.68$ seconds (4 pallets), $\overline{t_c}^* = 14$ seconds (28 pallets), the approximate optimal travel time $\overline{ESC}^* = 14.24$ seconds, with a real rack capacity of 2016 pallets; (2) for the 20%/90% ABC curve, $\overline{t_h}^* = 8.5$ seconds (17 pallets), $\overline{t_v}^* = 8.68$ seconds (4 pallets), $\overline{t_c}^* = 15$ seconds (30 pallets), the approximate optimal travel time $\overline{ESC}^* = 4.64$ seconds, and the real rack capacity is 2040 pallets. From the above results we find that the deviation of the approximate optimal solutions from the optimal solutions is fairly small: the deviation percentages (i.e. $(\overline{ESC}^* - ESC^*)/ESC^* \times 100\%$) are 0.27% and 0.66% respectively. Note that the resulting rack dimensions do not differ much. This is a phenomenon also known from 2-dimensional rack layouts: it is possible to find robust layouts good for various ABC-curves.

7. Conclusion

In this paper we discuss the design of a 3D (multi-deep) compact AS/RS system using a full turnover-based storage policy, which originates from the Distrivaart project. We extend the results from Le-Duc et al. (2005), based on the random storage policy, to the full turnover-based storage policy and find the optimal ratio of the three rack dimensions minimizing the expected single-command travel time of the S/R machine. From the results of the paper, we find that

(1) The optimal ratio between the three dimensions $t_c^* : t_h^* : t_v^*$ is independent of the rack's storage capacity V, but varies with the skewness parameter δ of the ABC curve. For a decreasing δ, or increasing turnover frequency for a given percentage of the inventoried items in the rack inventory, the optimal ratio t_c^* / t_h^* or t_c^* / t_v^* will increase. The problem with the random storage policy discussed by Le-Duc et al. (2005) is a special case of our problem with skewness parameter $\delta = 1$.

(2) For the 3-dimemsional rack system, the expected travel time will be minimized only when the rack is SIT in horizontal and vertical directions, which is similar to the results of Bozer and White (1984) and Le-Duc et al. (2005), but not cubic in time with any $s \in (0,1]$.

(3) The full turnover-based storage policy is a good assignment rule for improving the performance of the expected travel time of S/R machine for single command cycle. The more skewed (smaller δ) the ABC curve is, the more expected time is saved compared to the random storage policy. For example, for $\delta = 0.07$,(a 20%/90% ABC curve), the saved time is 67.68%.

(4) From Section 6, it can be seen that the optimal results for our discussed continuous 3D AS/RS are helpful to find an approximate optimal solution for practical examples.

The results in this paper may be extended in several directions, albeit the analysis may become cumbersome. It is interesting to study the class-based storage assignment.

Although class-based storage is not optimal, it is easier to implement in practice while it still can improve travel time substantially, compared to random storage. Second, the impact of dwell point strategies of the S/R machine can be studied. Third, multiple commands for a single cycle may be considered to improve the performance of the 3D AS/RS. Finally, the time needed for pickup/drop-off a pallet may also be considered in a 3D rack system although it is commonly omitted by researchers in 2D systems.

Appendix A. Calculation of Equation (5)

Because $W = \max\{t_h, t_v, t_c\}$ and $0 < b \le a \le 1$, the calculation of $y(j)$ should be classified into three cases (see Figure 4).

<Insert Figure 4 here>

Case 1: Let $j \le b^3/(ab) = b^2/a$; or $0 < W \le b$. Consider a location (x, y, z) in the jth fractile or percentile of the distance distribution (region A in Figure 4). By definition *j*% of the locations are closer to the I/O point than the location under consideration. These *j*% locations must be arranged in a cube in time, since the total time taken by the crane and conveyor to move from the I/O point to the P/D position of any point (x, y, z) is max(*x*,*y*,z). Since the dimension of the total warehouse is *ab*, the volume of this cube is j by the total volume ab, or abj. Therefore, for $j \le b^2/a$, the travel time from the depot to the location *j*th percentile is

$$y(j) = (abj)^{1/3}. \tag{17}$$

Case 2: Any point (x, y, z) located in region B (Figure 4) satisfies the location percentile $j \ge b^3/(ab) = b^2/a$ and $j \le a^2b/(ab) = a$; or $b \le W \le a$ where $W = \max(x, z)$. The racked locations of the jth percentile must be arranged in a rectangular block in time with $\sqrt{aj} \times b \times \sqrt{aj}$ in the horizontal, vertical and depth dimensions (so that the total volume is abj and $\sqrt{aj} \geq b$). Then, for $b^2/a \leq j \leq a$, the travel time from the depot to jth percentile location is

$$y(j) = \sqrt{aj} \,. \tag{18}$$

Case 3: Any point (x, y, z) located in the region C (Figure 4) satisfies $a \leq j \leq 1$ or $a \leq W \leq 1$ where $W = z$. The locations of jth percentile must be arranged in a rectangular block in time with $a \times b \times j$ in the horizontal, vertical and depth dimensions (so that the total volume is abj). Then for $a \leq j \leq 1$, the travel time from the depot to the location jth percentile is

$$y(j) = j \,. \tag{19}$$

Considering (17), (18) and (19), Equation (5) is obtained.

Appendix B. Proof of Theorem 1

Because the optimal solution (a,b) of model (12) is (a_s, b_s), its objective function value is

$$\begin{aligned} ESC_{CS}^{*}(a_s,b_s) = \frac{V^{1/3} a_s^{-s} s}{(1+s)(1+2s)(1+3s)(a_s b_s)^{1/3}} (2a_s^{\,s} + 2a_s^{\,1+2s} + b_s^{\,1+2s} \\ + 10a_s^{\,s} s + 6a_s^{\,1+2s} s + b_s^{\,1+2s} s + 12a_s^{\,s} s^2) \end{aligned} \,. \tag{20}$$

The constraint is the same for Models (11) and (12), so (a_s, b_s) is a feasible solution of model (11), and its corresponding objective function value of model (11) is

$$\begin{aligned} ESC_{CM}(a_s,b_s) = \frac{V^{1/3} a_s^{-s} s}{(1+s)(1+2s)(1+3s)(a_s b_s)^{1/3}} (2a_s^{\,s} + a_s^{\,1+2s} + a_s^{\,s} b_s^{\,1+s} \\ + b_s^{\,1+2s} + 10a_s^{\,s} s + 3a_s^{\,1+2s} s + 3a_s^{\,s} b_s^{\,1+s} s + b_s^{\,1+2s} s + 12a_s^{\,s} s^2) \end{aligned} \,. \tag{21}$$

From Equations (20) and (21), we have

$ESC_{CS}^{*}(a_s,b_s) - ESC_{CM}(a_s,b_s)$

$$=\frac{V^{1/3}a_s^{-s}s}{(1+s)(1+2s)(1+3s)(a_sb_s)^{1/3}}(a_s^{1+2s}-a_s^{s}b_s^{1+s}+3a_s^{1+2s}s-3a_s^{s}b_s^{1+s}s). \quad (22)$$

Because a,b,s, and $V>0$, we have

$$\frac{V^{1/3}a_s^{-s}s}{(1+s)(1+2s)(1+3s)(a_sb_s)^{1/3}}>0. \quad (23)$$

When $b_s=a_s$ for a and b in Models (11) and (12), $a_s^{1+2s}-a_s^{s}b_s^{1+s}=0$ and $3a_s^{1+2s}s-3a_s^{s}b_s^{1+s}s=3a_s^{s}s(a_s^{1+s}-b_s^{1+s})=0$. Considering Equations (23) and (22), we have $ESC_{CS}^{*}(a_s,b_s)-ESC_{CM}(a_s,b_s)=0$. If (a_s,b_s) equals (a_m,b_m) and $b_s=a_s$, then (a_s,b_s) is also the optimal solution of Model (11) and $ESC_{CM}^{*}(a_m,b_m)=ESC_{CM}(a_s,b_s)$. In this case $ESC_{CM}^{*}(a_m,b_m)=ESC_{CS}^{*}(a_s,b_s)$ holds. (I) is proven.

Otherwise, $b_s<a_s$, in Equation (22), $a_s^{1+2s}-a_s^{s}b_s^{1+s}=a_s^{s}(a_s^{1+s}-b_s^{1+s})>0$ and $3a_s^{1+2s}s-3a_s^{s}b_s^{1+s}s=3a_s^{s}s(a_s^{1+s}-b_s^{1+s})>0$. Considering Equations (23) and (22), we have $ESC_{CS}^{*}(a_s,b_s)-ESC_{CM}(a_s,b_s)>0$. Because (a_m,b_m) is the optimal solution of Model (11), for the minimized model, $ESC_{CM}^{*}(a_m,b_m)\le ESC_{CM}(a_s,b_s)$. Thus $ESC_{CS}^{*}(a_s,b_s)-ESC_{CM}^{*}(a_m,b_m)>0$. (II) is proven.

Appendix C. Proof of Theorem 3

From the above Theorems 1-2, we know the optimal result of Model GM exists in the case CL, where the conveyor's length is the longest. Thus, if we can prove the 3D rack must be SIT for ESC_{CL} to be minimized, then Theorem 3 is proven. For convenience, we use an equivalent version of the model, different from Model (10) only in form, for the case CL as follows:

$$\begin{aligned} &Minimize \;\; ESC_{CL}(a,b,t_c)=\frac{a^{-s}st_c}{(1+s)(1+2s)(1+3s)}(a^s+a^{1+s}+a^{1+2s}+b^{1+s} \\ &\qquad +b^{1+2s}+5a^s s+5a^{1+s}s+3a^{1+2s}s+3b^{1+s}s+b^{1+2s}s+6a^s s^2+6a^{1+s}s^2) \\ &subject\ to \quad abt_c^3=V \\ &\qquad\qquad 0<b\le a\le 1 \\ &\qquad\qquad t_c>0. \end{aligned} \tag{24}$$

We use reduction to absurdity to prove that the optimal 3D rack is SIT when ESC_{CL} is minimized.

Suppose the optimal rack were not SIT for CL. Let $(\overline{a},\overline{b},\overline{t}_c)$ and $\overline{ESC}_{CL}$ denote the optimal solution and objective function value of Model (24). Then we have $\overline{a}>\overline{b}$, and $\overline{ESC}_{CL}\le ESC_{CL}$.

Because $abt_c{}^3=V$, we have $\overline{a}\overline{b}=V/\overline{t}_c^3$. Let $\overline{a}\overline{b}=V/\overline{t}_c^3=k$ (k is a positive constant). We can design a new solution: $a=b=k^{1/2}$, and $t_c=\overline{t}_c=\sqrt[3]{V/(\overline{a}\overline{b})}$ $=\sqrt[3]{V/k}$ that provides the 3D SIT rack.

Then we obtain

$$\begin{aligned} &\overline{ESC}_{CL}(\overline{a},\overline{b},\overline{t}_c)-ESC_{CL}(k^{1/2},k^{1/2},\overline{t}_c)=\frac{\overline{a}^{-(1+3s)}sV^{1/3}}{(1+s)(1+2s)(1+3s)k^{1/3}}(\overline{a}^{2+3s} \\ &\quad +\overline{a}^{2+4s}-2\overline{a}^{1+3s}\sqrt{k}-2\overline{a}^{1+3s}\sqrt{k^{1+s}}+\overline{a}^s k^{1+s}+k^{1+2s}+5\overline{a}^{2+3s}s+3\overline{a}^{2+4s}s \\ &\quad -8\overline{a}^{1+3s}s\sqrt{k}-4\overline{a}^{1+3s}s\sqrt{k^{1+s}}+3\overline{a}^s k^{1+s}s+k^{1+2s}s+6\overline{a}^{2+3s}s^2-6\overline{a}^{1+3s}s^2\sqrt{k}). \end{aligned} \tag{25}$$

Because $\overline{a},s,k$ and $V>0$, we have $\frac{\overline{a}^{-(1+3s)}sV^{1/3}}{(1+s)(1+2s)(1+3s)k^{1/3}}>0$. Define

$$\begin{aligned} f(x)=&\ x^{2+3s}+x^{2+4s}-2x^{1+3s}\sqrt{k}-2x^{1+3s}\sqrt{k^{1+s}}+x^s k^{1+s}+k^{1+2s}+5x^{2+3s}s+3x^{2+4s}s \\ &-8x^{1+3s}s\sqrt{k}-4x^{1+3s}s\sqrt{k^{1+s}}+3x^s k^{1+s}s+k^{1+2s}s+6x^{2+3s}s^2-6x^{1+3s}s^2\sqrt{k}. \end{aligned} \tag{26}$$

Then to check $\overline{ESC}_{CL}(\overline{a},\overline{b},\overline{t}_c)-ESC_{CL}(k^{1/2},k^{1/2},\overline{t}_c)<0$ or not is equivalent to check $f(\overline{a})<0$ where $\overline{a}\ne k^{1/2}$.

With x,s and $k>0$, we have

$$\frac{\partial^2 f(x)}{\partial x^2} = \frac{x^{-3(1+s)} s}{1+2s} (2x^s k^{1+s} + 2k^{1+2s} + x^{2(1+2s)} s + 4x^s k^{1+s} s + 3k^{1+2s} s) > 0 \qquad (27)$$

Equation (27) shows that $f(x)$ is a strictly convex function of x. Therefore at most one critical point exists and satisfies

$$\frac{\partial f(x)}{\partial x} = \frac{x^{-3(1+s)} s}{1+2s} (x^{2+3s} + x^{2+4s} - x^s k^{1+s} - k^{1+2s} + 2x^{2+3s} s + x^{2+4s} s - 2x^s k^{1+s} s - k^{1+2s} s) = 0 \,. \qquad (28)$$

If the point exists, the corresponding value of $f(x)$ must be the overall minimal point. Set $x = \sqrt{k}$, we find that $\frac{\partial f(x)}{\partial x}\Big|_{x=\sqrt{k}}$ =0 and $\sqrt{k}$ is the critical point. That is, min $f(x) = f(\sqrt{k}) = 0$. Then we have $f(x) > 0$ for all $x \neq \sqrt{k}$.

Because the optimal rack were not SIT for CL (i.e. $\overline{a} > \overline{b}$), and $\overline{a}\,\overline{b} = k > 0$, we have $\overline{a} > \sqrt{k} > \overline{b}$ (here $\overline{a} \neq \sqrt{k}$), and then $f(\sqrt{k}) < f(\overline{a})$. Then $f(\overline{a}) > 0$, which implies that $\overline{ESC}_{CL}(\overline{a}, \overline{b}, \overline{t}_c) - ESC_{CL}(k^{1/2}, k^{1/2}, \overline{t}_c)$ >0, contradicting that $(\overline{a}, \overline{b}, \overline{t}_c)$ is the optimal solution of Model (24). Hence, we have completed the proof of Theorem 3.

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List of the Tables and Figures

Figure 1: Distrivaart: A conveyor-supported automated compact storage system on a barge (source: Waals, 2005).

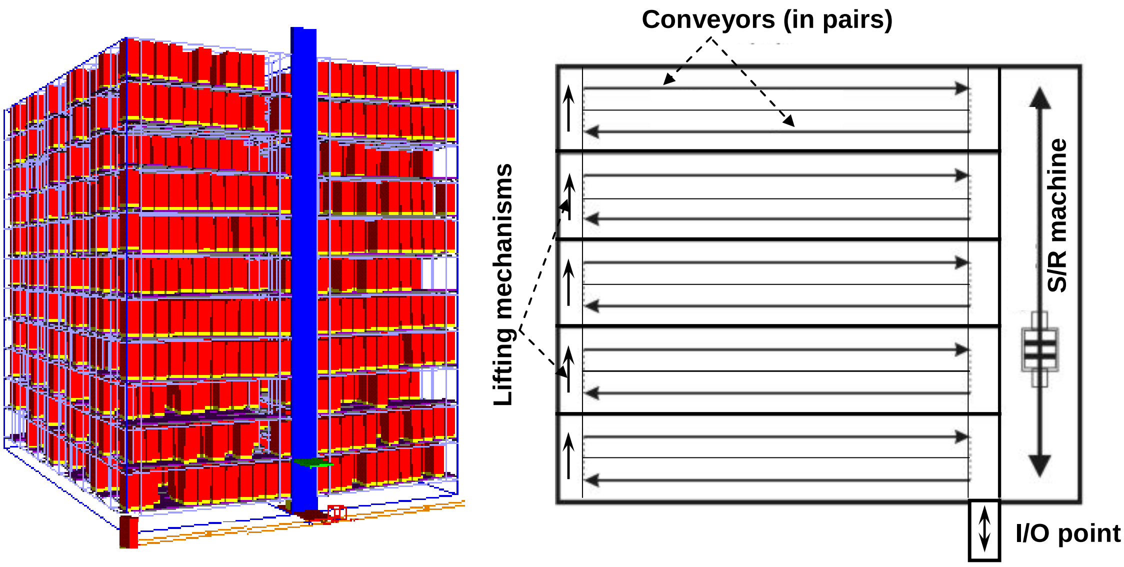

Figure 2: A compact S/RS with gravity conveyors for the depth movements (Le-Duc et al. 2005)

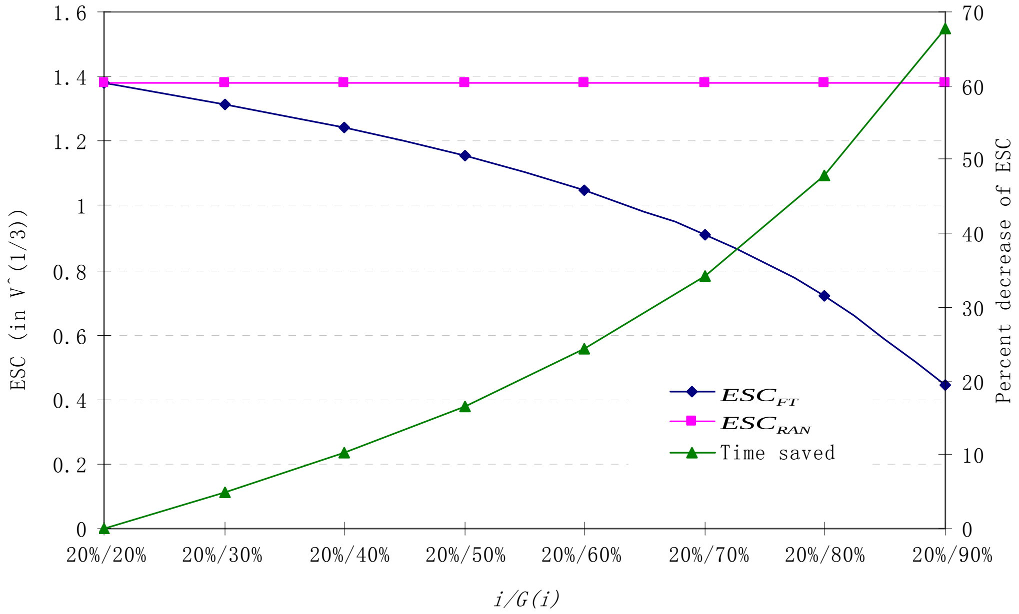

Figure 3: Reduction of ESC of the full turnover-based storage policy compared with that of the random storage for various $i/G(i)$

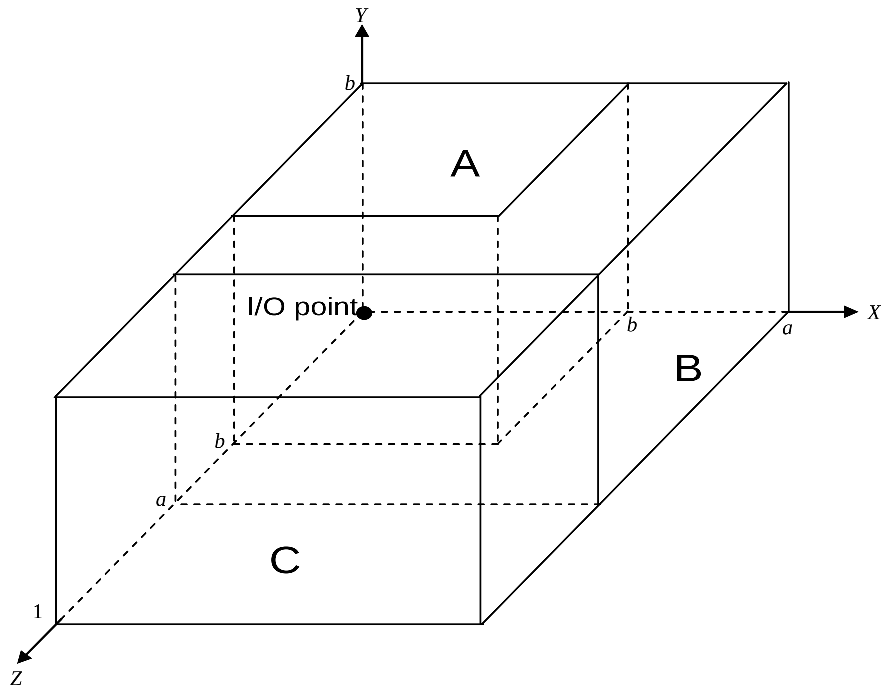

Figure 4: Three storage regions in the 3D rack

Table 1: The optimal solutions for different skewness parameters(ABC curves)

δ	s	ABC Curve	a^*	b^*	t_h^*	t_v^*	t_c^*	ESC^*
1.00	1.00	20%/20%	0.72	0.72	0.90	0.90	1.24	1.38
0.75	0.86	20%/30%	0.70	0.70	0.89	0.89	1.27	1.31
0.57	0.73	20%/40%	0.68	0.68	0.88	0.88	1.29	1.24
0.43	0.60	20%/50%	0.66	0.66	0.87	0.87	1.31	1.15
0.32	0.48	20%/60%	0.64	0.64	0.86	0.86	1.35	1.05
0.22	0.36	20%/70%	0.61	0.61	0.85	0.85	1.38	0.91
0.14	0.24	20%/80%	0.58	0.58	0.84	0.84	1.43	0.72
0.07	0.12	20%/90%	0.55	0.55	0.82	0.82	1.50	0.45

Table 2: System parameters

Total system capacity (V)		2000 pallets
Storage policy		Full turnover-based storage
Pallet size in seconds (width x length x height)	Net	0.4 x 0.4 x 2
	Gross	0.5 x 0.5 x 2.17
S/R machine	Operating policy	Single-command cycle
	Vertical speed (s_v)	0.8 (meter per second)
	Horizontal speed (s_h)	2.8 (meter per second)
Conveyor speed (s_c)		1.6 (meter per second)
Cases of ABC curve considered		20%/20% and 20%/90%

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